Suggested solutions to the IO (BSc) exam on August 15, 2011
VERSION: September 6, 2011

Question 1

Part a)

There is a unique Nash equilibrium of the game: each firm charges a price that equals its marginal cost, $(p_1^*, p_2^*) = (c, c)$. The students should of course prove their claims (the calculations/arguments that are needed are standard — see textbook or lecture notes).

Part b)

[To the external examiner: This model is identical to one in Tirole and in the lecture slides.]

- *Claim that we are asked to prove:* the following is a Nash equilibrium: Both firms charge the price

$$\boxed{p^* = 1 - \overline{q}_1 - \overline{q}_2.}$$

  - *Interpretation of the claim*: It is as if the firms dumped their full production capacities on the market, and then a "Walrasian auctioneer" chose a price that made sure that supply is equal to demand.

Proof of the claim

- We need to show that a firm cannot, given that the rival charges p^*, increase its profit by either choosing a price $p < p^*$ or a price $p > p^*$.
- First note that if both firms charge the price p^*, each of them earns a positive profit:

$$\boxed{\Pi_1 = p^* \overline{q}_1 > 0} \qquad \text{and} \qquad \boxed{\Pi_2 = p^* \overline{q}_2 > 0.}$$

- If charging a *lower* price ($p < p^*$), firm 1 would be able to sell more. But since it is already operating at its capacity constraint it cannot produce more, so a lower price would not increase its profit.
- Could the firm gain by *increasing* its price? If charging a higher price ($p > p^*$), firm 1's profit would equal:

$$\boxed{\Pi_1(p) = p(1 - p - \overline{q}_2)}$$

  (here we use the assumption of efficient rationing—see also the attached figure. [IO-2009-L5-I, fig 6]).

- Differentiate w.r.t. p:

$$\boxed{\frac{\partial \Pi_1(p)}{\partial p} = 1 - 2p - \overline{q}_2.}$$

- Evaluate at $p = p^*$ $(\equiv 1 - \overline{q}_1 - \overline{q}_2)$:

$$\boxed{\frac{\partial \Pi_1(p)}{\partial p}\,|_{p=p^*} = 1 - 2\overbrace{(1 - \overline{q}_1 - \overline{q}_2)}^{=p^*} - \overline{q}_2 = 2\overline{q}_1 + \overline{q}_2 - 1 \leq 0.}$$

- That is: increasing p, starting at p^*, would not raise profits.

- We have thus shown that given that the rival firm charges p^*, a firm cannot increase its profit by increasing or decreasing its price from p^*. This means that $(p_1, p_2) = (p^*, p^*)$ is a Nash equilibrium, which we were asked to prove.

Part c) *Explain (i) what kind of model Kreps and Sheinkman (Bell Journal of Economics, 1983) studied and (ii) what result they could show. Also, (iii) discuss the limitations and implications of their analysis and result.*

- (i) Kreps and Scheinkman studied a two-stage game where the firms, in the first stage, simultaneously choose capacities $\overline{q}_i$ (at some cost). Then at stage 2, knowing each other's capacity, the firms simultaneously choose prices p_i.

- (ii) The result that they could show can be summarized as follows:

  - Suppose the demand function is concave and the rationing rule is the efficient one.
  - Then the outcome (i.e., the equilibrium capacities/quantities and the equilibrium price) of the two-stage game is the same as that of the corresponding one-stage Cournot game.

- (iii) The result is a celebrated one and many economists interpret it as a justification for thinking of Cournot games as a reduced form representation of the two-stage game described above. This is appealing, because the story in the two-stage game sounds plausible and realistic (in particular, in that story there is someone who actually sets the prices, in contrast to the Cournot model). At the same time, the outcome is not as unrealistic as in the Bertrand model, where the equilibrium involves marginal cost pricing even when there are only two firms. So the outcome of the two-stage game combines the good and appealing features of the Bertrand and Cournot models, while avoiding the drawbacks with each of those models. However, there are some caveats:

  - The result obtained by Kreps and Scheinkman, which can be referred to as "*Cournot outcome in the two-stage game*", is **weaker** than our result under a) where we obtained the "*exact Cournot reduced form*".

With the latter result, we actually get exactly the Cournot profit functions, $\Pi_i^{net} = [1 - (\overline{q}_1 + \overline{q}_2)]\,\overline{q}_i - c_0\overline{q}_i$ (where c_0 is the investment cost). This means that we in that case can also study a version of the Cournot model with, for example, sequential quantity choices.

– The Kreps-Sheinkman result is not very robust to changes in the assumptions. For example, it relies critically on the assumption of the efficient-rationing rule.

– In more general settings, the capacity choices in the full game may serve important roles that are not captured by a reduced form. For example, firms with **private information** may want to use the capacity choices as **informative signals** to its rivals.

- A summary of Tirole's discussion of the implications of Kreps-Scheinkman's result:

  – The predictions and welfare results of the traditional Cournot model can be provided with foundations in some extreme cases.

  – The two-stage game illustrates a broad idea that firms may want to choose non-price actions that soften price competition.

  – In many applications the exact Cournot profit functions are not essential. Instead the key thing is that the best-response functions are downward-sloping—i.e., that the firms' choice variables are **strategic substitutes**:

$$\boxed{\frac{\partial^2 \Pi_i}{\partial \overline{q}_i \partial \overline{q}_j} = \frac{\partial^2\left(\left[P\left(\overline{q}_i + \overline{q}_j\right) - c_0\right]\overline{q}_i\right)}{\partial \overline{q}_i \partial \overline{q}_j} = P' + P''\overline{q}_i < 0.}$$

  This may very well hold even if the "*exact Cournot reduced form*" does not hold (Kreps-Sheinkman assumed $P'' \leq 0$).

Question 2

[To the external examiner: This model is a tweaked version of a model in Tirole and in the lecture slides.]

Part a)

- From the question we have that firm 1's full information demand equals

$$\overline{\theta} = \frac{p_2 - p_1 + t}{2t}. \tag{1}$$

  - Firm 2's full information demand is therefore

$$1 - \overline{\theta} = \frac{p_1 - p_2 + t}{2t}.$$

- We also have from the question that the fraction of consumers who are reached by the ad from at least one of the firms equals

$$1 - (1 - \Phi_1)(1 - \Phi_2) = \Phi_1 + \Phi_2 - \Phi_1\Phi_2. \tag{2}$$

- Hence firm 1's actual demand (i.e., the number of people who know about the firm's existence and also chooses to buy from it), given advertising levels Φ_1 and Φ_2, is given by the product of (1) and (2):

$$D_1 = (\Phi_1 + \Phi_2 - \Phi_1\Phi_2)\left(\frac{p_2 - p_1 + t}{2t}\right).$$

  - Similarly, for firm 2 we have

$$D_2 = (\Phi_1 + \Phi_2 - \Phi_1\Phi_2)\left(\frac{p_1 - p_2 + t}{2t}\right).$$

Part b)

- Firm 1's problem is to maximize

$$\begin{aligned}\Pi^1 &= (p_1 - c) D_1 - \frac{a}{2}\Phi_1^2 \\ &= (\Phi_1 + \Phi_2 - \Phi_1\Phi_2)\left(\frac{p_2 - p_1 + t}{2t}\right)(p_1 - c) - \frac{a}{2}\Phi_1^2\end{aligned}$$

  with respect to p_1 and Φ_1.

- FOC w.r.t p_1:

$$\frac{\partial \Pi^1}{\partial p_1} = (\Phi_1 + \Phi_2 - \Phi_1\Phi_2)\left[-\frac{1}{2t}(p_1 - c) + \left(\frac{p_2 - p_1 + t}{2t}\right)\right] = 0 \Leftrightarrow$$

$$p_1 - c = p_2 - p_1 + t.$$

- FOC w.r.t. Φ_1:

$$\frac{\partial \Pi^1}{\partial \Phi_1} = (1-\Phi_2)\left(\frac{p_2 - p_1 + t}{2t}\right)(p_1 - c) - a\Phi_1 = 0 \Leftrightarrow$$

$$(1-\Phi_2)\left(\frac{p_2 - p_1 + t}{2t}\right)(p_1 - c) = a\Phi_1.$$

- Consider again the p_1-FOC, impose symmetry and solve for p:

$$p - c = p - p + t \Rightarrow p = c + t. \tag{3}$$

- Consider again the Φ_1-FOC, impose symmetry and solve for Φ:

$$(1-\Phi)\left(\frac{p - p + t}{2t}\right)(p - c) = a\Phi \Rightarrow (1-\Phi)\overbrace{(p-c)}^{=t} = 2a\Phi \Rightarrow \Phi = \frac{t}{2a+t},$$

  where the result in (3) has been used.

- Summing up, the equilibrium price for each firm is

$$\boxed{p = c + t,}$$

  and the equilibrium advertising level for each firm equals

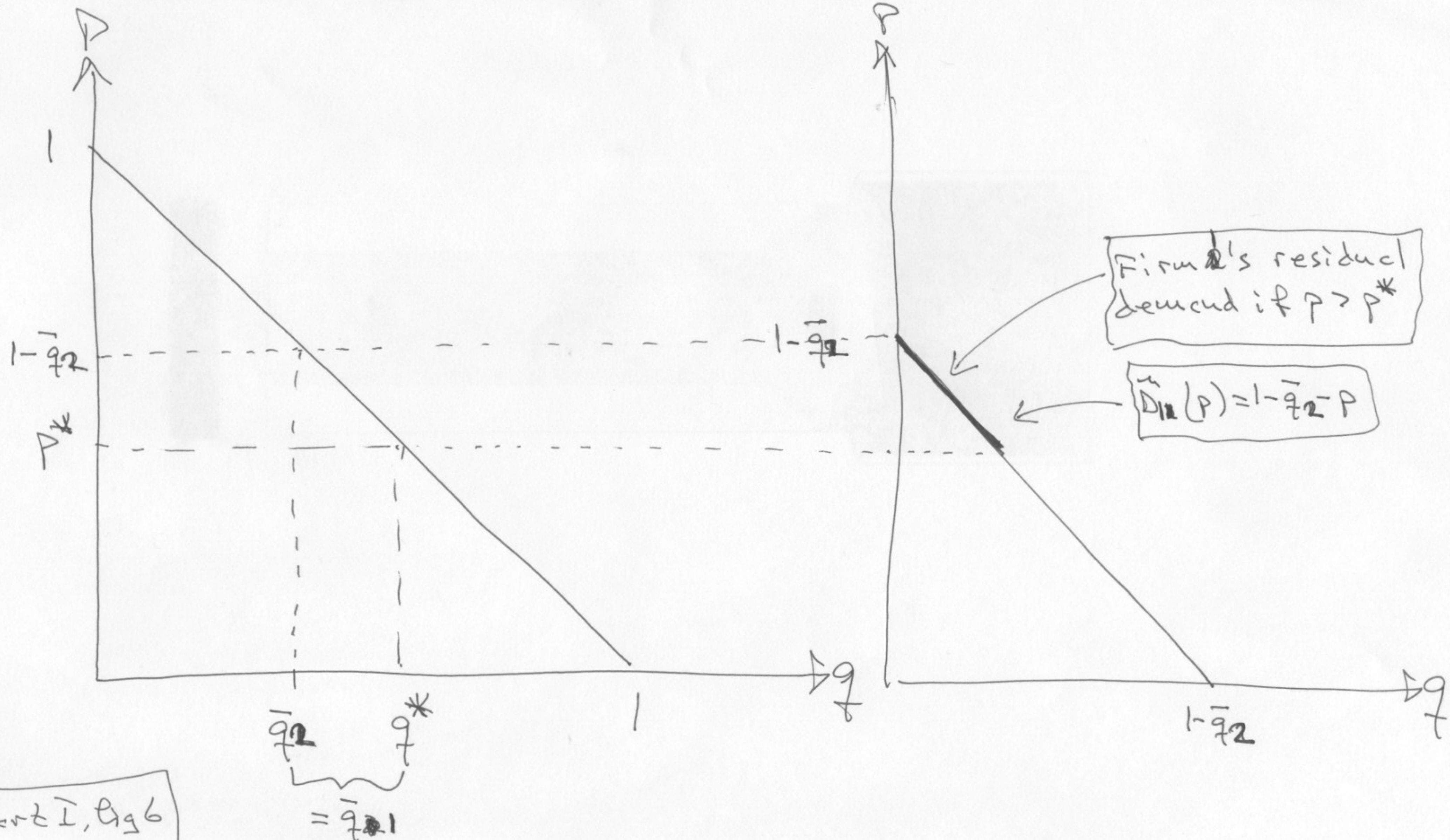
$$\boxed{\Phi = \tfrac{t}{2a+t}.}$$

Part c)

- Quote from Tirole, page 293:

  What is more remarkable, [equilibrium profits] increase with the cost of advertising. The direct effect of an increase in a (for p and Φ given) is to reduce the firms' profits. However, there is a strategic effect: An increase in advertising costs reduces advertising and thus increases informational product differentiation. This allows the firms to raise the price. In this example, they gain more from costlier advertising than they lose. This result is not general, but it strongly exemplifies the role of advertising in reducing product differentiation. It may also shed some light on why some professions do not resist—and sometimes encourage—legal restrictions om advertising.

Firm 1's residual demand if charging $p > p^*$ (given efficient rationing)



LS-Part I, Fig 6